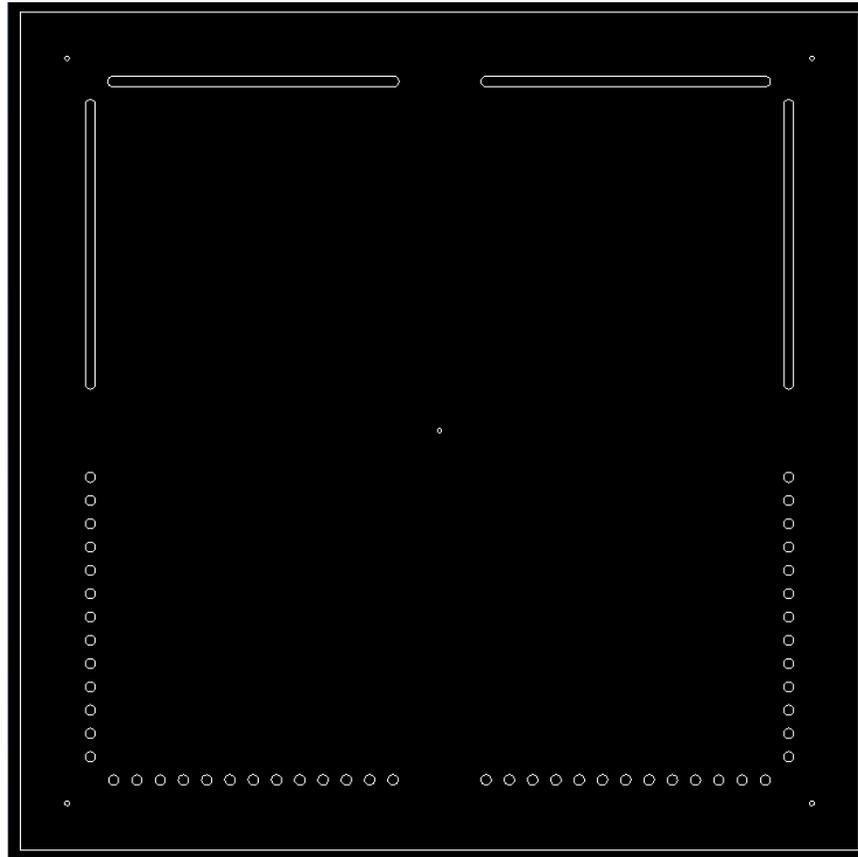


1. I decided to build a very utilitarian planar exact constraint model. My design has 4 different regions, two that allow infinite adjustability of the pegs, and two that have a finite number of slots for the pegs to be inserted into. The reason I chose to have two of each type is that I wanted to explore the effect of the peg-object friction on its constraint. To get very different coefficients of friction I used aluminum pegs on two regions and rubber pegs on the other two. The thought (which turned out to be true) was that the rubber pegs would increase the range of force vectors that would remain constrained (by making slippage along the pegs more difficult).



*Figure 1: CAD Drawing of Planar Exact Constraint Model*

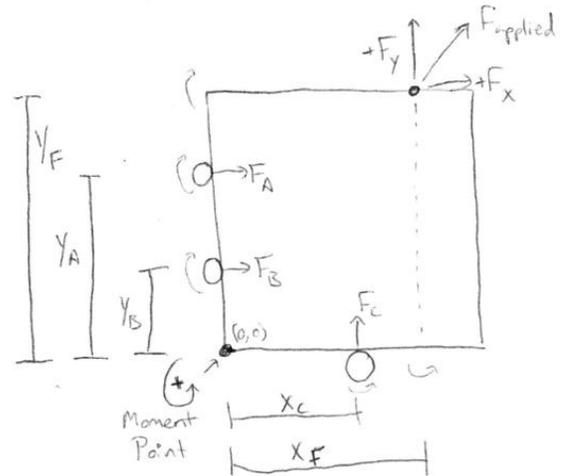
2. Here is the math for my model, which is a simple one that does not incorporate friction (as I was already at the limit of my statics know-how). It was the product of much thinking and some great advice and help from my peer review partners Nick and Sally.

$$F_y + F_c = 0$$

$$F_x + F_A + F_B = 0$$

$$\sum M_{(0,0)} = 0 = F_c x_c + F_y x_f - F_B y_B - F_A y_A - F_x y_f$$

$y_f, y_A, y_B, x_c, x_f, F_x,$  and  $F_y$  are known, solving for  $F_A, F_B, F_c$



- I implemented my model using Mathematica. Mathematica's "Manipulate" tool allows a user to easily change one or more inputs to an arbitrarily complex system and have the system's output dynamically updated. The first two inputs in my model define the applied force vector. The ranges allow the user to explore the full range of relevant input forces. The last three inputs allow the user to adjust the position of the three pegs that form the EC system. The outputs shown in the system graphic are the force vector and its line of action, the angle and XY components of the applied force, and the reaction (normal) force from each of the pegs. The color of the square dynamically changes from green (good constraint) to yellow (poor constraint due to applied force components moving away from the peg positions ( $90^\circ < \text{angle} < 0^\circ$ )) to red (the applied force will cause the square to tip (pivot around a peg)). The Mathematica files for the model (and a video showing its dynamic operation) are included as links on my webpage.
- To build my physical model I used a laser cutter to cut the holes and slots in a piece of ~0.2" thick acrylic. Before cutting my actual piece I made some test cuts with the laser cutter to establish the kerf for my material and cutting parameters. Once I had established that, I quickly modified my CAD so that I would get accurate dimensions. After cutting, I measured the dimensions and was reasonably satisfied with its accuracy (to within 0.05mm (0.002")). I then went ahead and ran a #34 reamer through the peg holes to make them a nice snug fit with a standard 4-40 screw. I also tapped the 5 mounting holes and affixed 5 rubber mounting feet (the same feet were used as my rubber "pegs").

For the high friction pegs, I used rubber mounting feet with a 12.7mm (0.5") OD and an inner hole sized for a 4-40 screw. For the low friction pegs, I quickly made 12.7mm (0.5") OD aluminum pegs on the lathe.

For the slot-based constraints, the pegs were secured to the slot using a 4-40 screw, a washer, and a nut. It was easy enough to tighten and loosen these by hand, but a hex key and 1/4" wrench makes the positions more permanent.

Here are some pictures of the finished model:

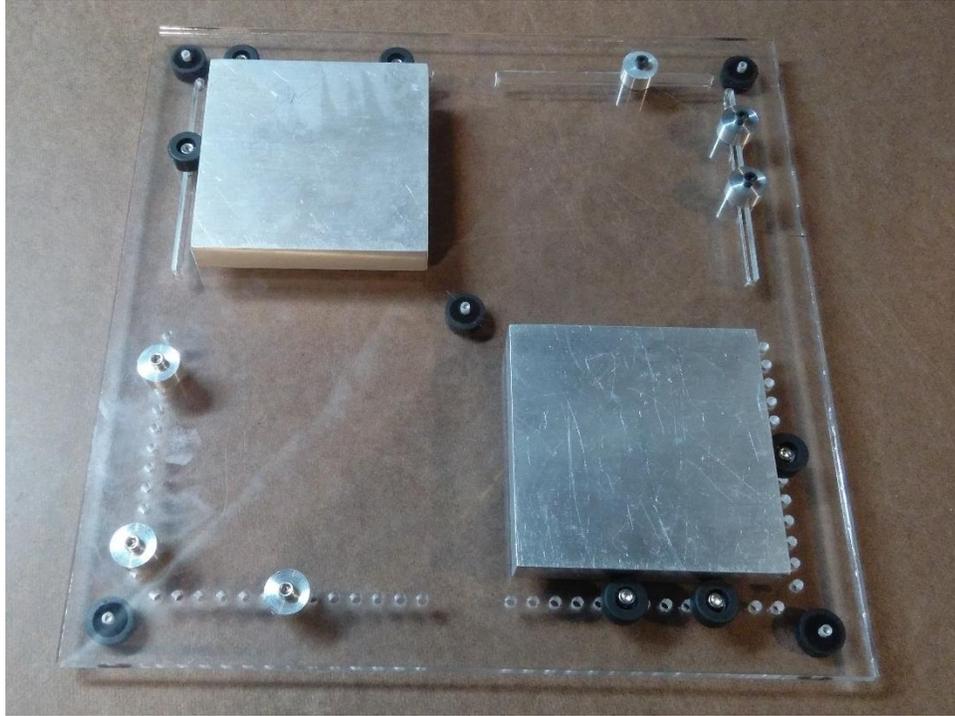


Figure 2: Finished Model with squares and pegs in position

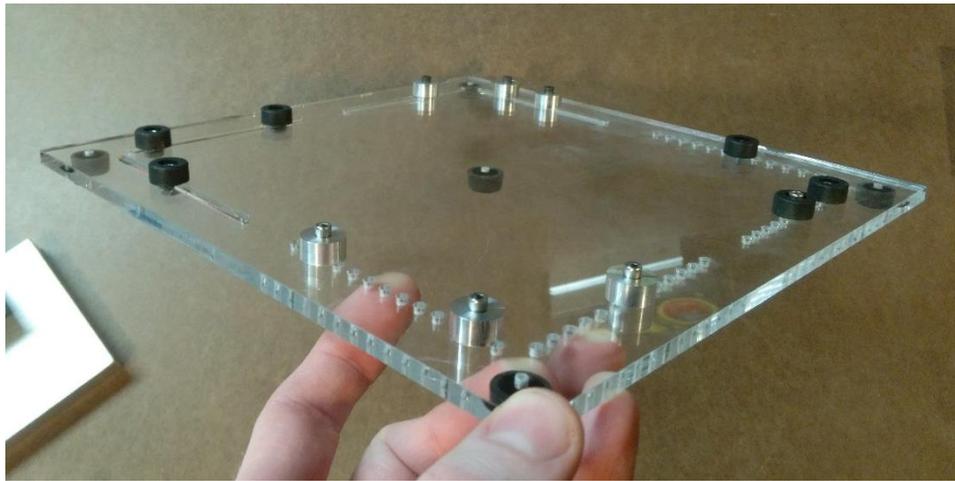
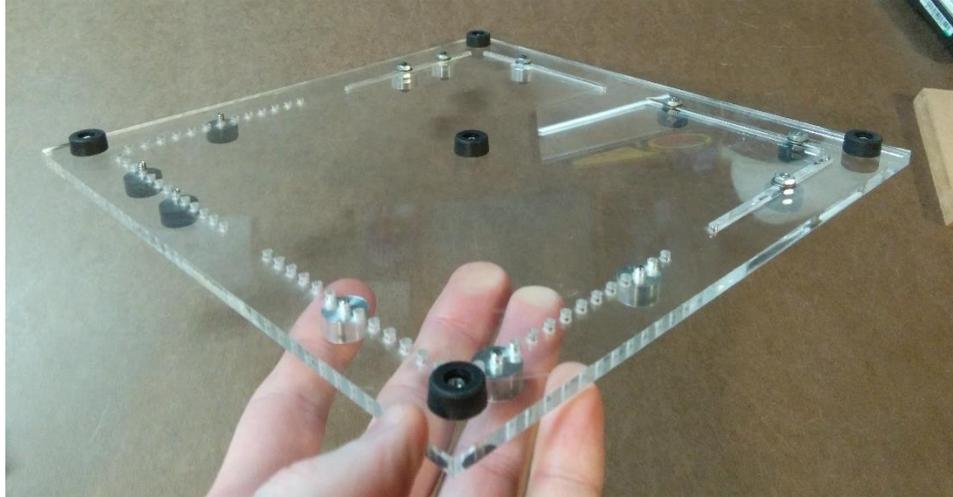


Figure 3: Side-ish view of the top



*Figure 4: Side-ish view of the bottom*

5. During our peer review meeting we played around with each other's models to explore their sensitivity. Nick's idea to use a rubber band and a variety of peg placements outside his EC system to simulate the applied force was very clever. Sally's model was a simple but effective demonstration of both good and bad peg placements for planar constraint. It was also definitely the most appealing from the "build a toy" aesthetic.